Specific Heat Capacity with Multiple Materials

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We know the specific heat capacity of water is 4182 J/KgK, we can use this to figure out the heat capacity of other materials too.

Let's add 30g of steel bolts to 50g of water and heat it over the same candle for a measured amount of time. Measure the temperature of the water + bolts mixture before and after. Can we use this to find the specific heat capacity of the steel?

Yes like this. The equation we use is

$$E = mc\Delta T$$

Let's write it out for the water and the steel:

$$E_w = m_w c_w \Delta T_w \quad E_s = m_s c_s \Delta T_s$$

Some key facts:

1. The total energy E is the sum of the energies in the water E_w and steel E_s.
2. ΔT is the same for the water and bolts because they're in close contact, good conductors and are being heated slowly.
3. E equals the candle power times the time it was heating the water and bolts. $E = Pt$.

Using the first and second facts we can write:

$$E = E_w + E_s = m_w c_w \Delta T + m_s c_s \Delta T$$

factorising

$$E = \Delta T \left(m_w c_w + m_s c_s\right)$$

Using the third fact and rearranging...

$$\frac{Pt}{\Delta T} = m_w c_w + m_s c_s.$$

Which is what we want! The only thing we don't know in this equation is c_s. Rearrange and substitute in.

Finally... This last equation can be rewritten really nicely since $E = Pt$ and from the usual specific heat capacity equation we have:

$$\frac{E}{\Delta T} = mc$$

the left side is simply mc

$$mc = m_w c_w + m_s c_s$$

Where m is the combines mass and c is the combined heat capacity of the water and steel bolts.

So in general a system in thermal equilibrium composed of multiple masses of different materials obeys:

$$mc = \sum_i m_i c_i$$